

**WORCESTER POLYTECHNIC INSTITUTE
TWENTY-SEVENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 21, 2014
TEAM EXAM QUESTION SHEET WITH ANSWERS**

DIRECTIONS: Please write your answers on the **TEAM ANSWER SHEET** provided. This part of the contest is 45 minutes. All 14 problems are counted equally. Calculators and other electronics **MAY NOT** be used.

1) Simplify $1 + i + i^2 + \dots + i^{100}$

$$i = \sqrt{-1}$$

Ans: 1

2) Simplify the following matrix product.

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{1203} \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Ans: (-3,3)

3) Find positive integer solutions to

$$a^3 - b^3 - c^2 = 3abc$$

$$a^2 = 2(b + c)$$

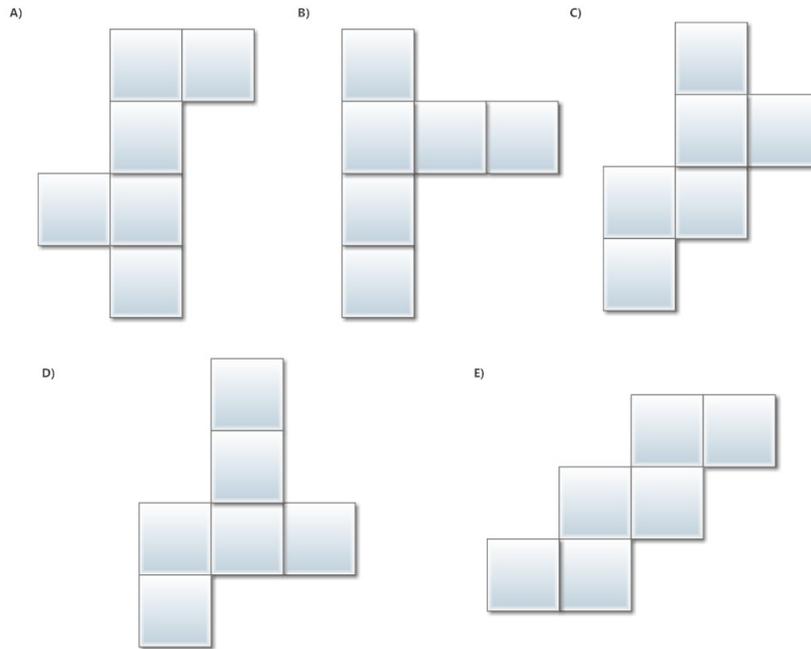
Ans: a=2, b=1, c=1

4) Solve for x.

$$(6x + 28)^{\frac{1}{3}} - (6x - 28)^{\frac{1}{3}} = 2$$

Ans: x = +/- 6

5) Which one **cannot** be folded into a cube?



Ans: B) cannot be

6) Solve the following system:

$$x + y + z + u = 5$$

$$y + z + u + v = 1$$

$$z + u + v + x = 2$$

$$u + v + x + y = 0$$

$$v + x + y + z = 4$$

Ans: $x=2, y=1, z=3, u=-1, v=-2$

7) A system of 2 linear equations and 2 unknowns have coefficients that are in arithmetic progression. What is the solution to the system?

Ans: -1 and + 2

8) John wants to buy \$1 worth of stamps. He asked for 2 cent stamps, ten times as many 1 cent stamps and the rest in 5 cent stamps. How many of each did he buy?

Ans: 50 one cent stamps, 5 two cent stamps, 8 five cent stamps

9) One of the following numbers is prime. They are displayed in binary form. Which one is prime?

- a) 10001000_2
- b) 10100110_2
- c) 100000010_2
- d) 101111011_2
- e) 100111000_2

Ans: d) is prime

10) Suppose 6 students are taking an exam in a row of seats, with an aisle at each end. If they finish the exam in random order, what is the probability that a student will have to pass over one or more of the other students in order to reach the aisle?

Ans: $1 - (2/6) * (2/5) * (2/4) * (2/3) = 43/45$

11) Evaluate

$$\log_{10} \cot(1^\circ) + \log_{10} \cot(2^\circ) + \log_{10} \cot(3^\circ) + \dots + \log_{10} \cot(87^\circ) + \log_{10} \cot(88^\circ) + \log_{10} \cot(89^\circ)$$

Ans: 0

12) What is the 2008th digit in the decimal representation of $1/14$?

Ans: 4

13) In a certain high school there are 605 students. There are 40 more sophomores than freshman and one half as many freshmen as juniors. The number of seniors is 30 less than 3 times the number of freshman. How many seniors are there?

Ans: 225 seniors

14) Solve for x.

$$(\ln(3x))^4 - (\ln(3x))^3 - 6(\ln(3x))^2 = 0$$

Ans: $x = 1/3, e^3/3, e^{-2}/3$